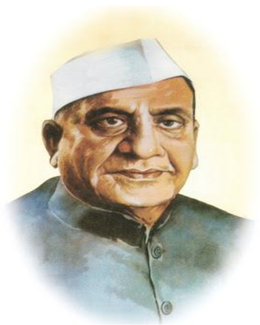


Mahatama Phule Arts, Commers, Science Mahavidyalaya, Warud

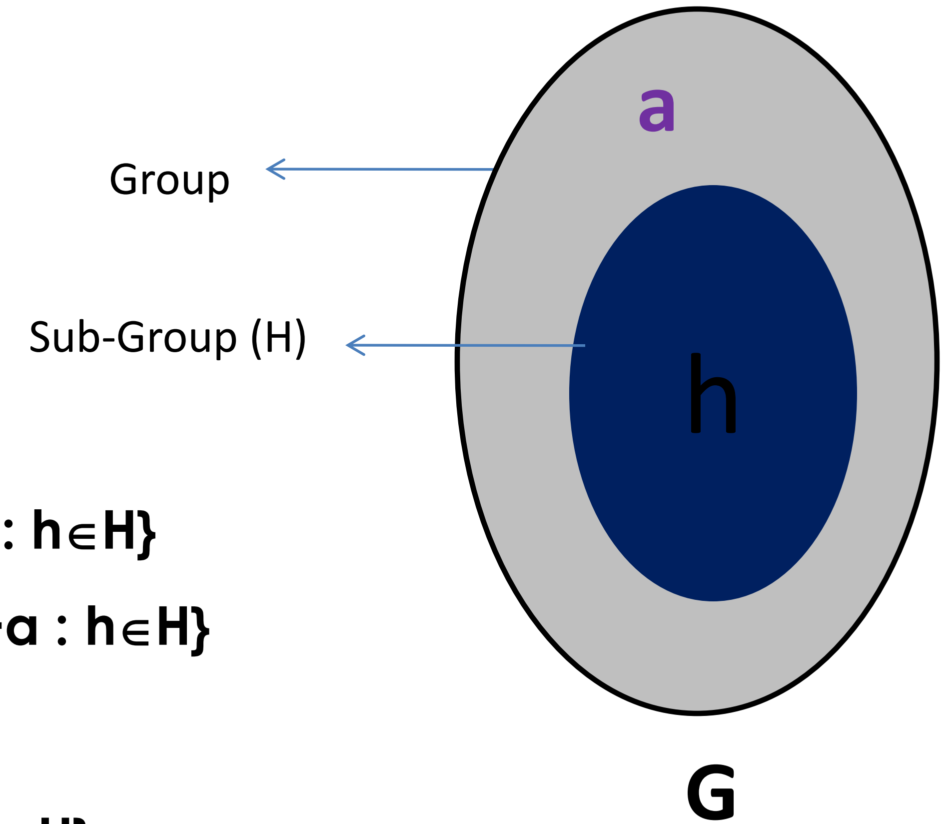
Department of Mathematics

Quotient Group

Dr. R. S. Wadbude
Associate Professor

Contents

Coset



1. Under Addition

a) Left Coset : $a+H=\{a+h : h\in H\}$

b) Right Coset : $H+a= \{h+a : h\in H\}$

2. Under Multiplication

a) Left Coset : $aH=\{ah : h\in H\}$

b) Right Coset : $Ha= \{ha : h\in H\}$

Normal Sub-Group

A sub-group N of G is called *normal Sub-group* of G if for every **g** $\in$ **G** and for every **n** $\in$ **N**, we have $gng^{-1} \in$ **N.**

Condition :

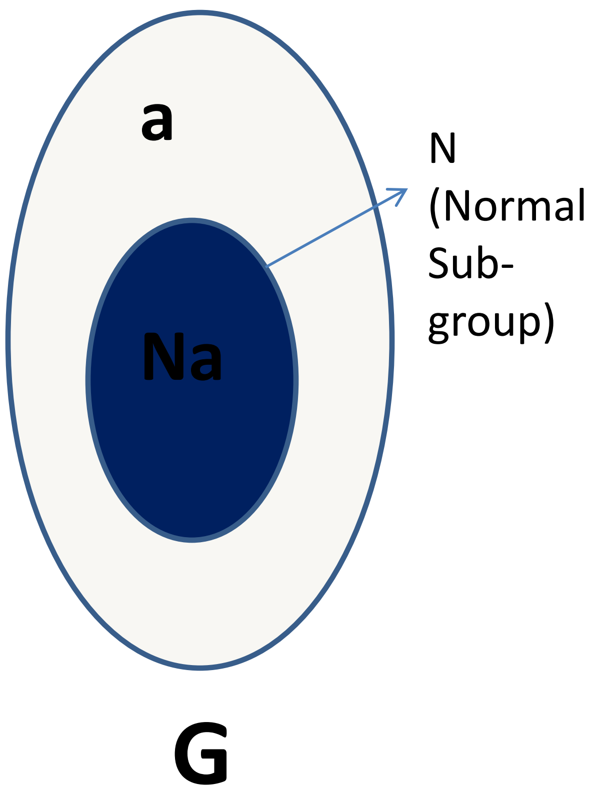
1. $gng^{-1} \in N$ $\forall\ g \in G, n \in N$
2. $gNg^{-1} \subset N$ $\forall\ g \in G$
3. $gNg^{-1} = N$ $\forall\ g \in G$
4. $gN = Ng$ $\forall\ g \in G$
5. $NaNb = Nab$ $\forall\ a, b \in G$

Quotient Group

- Let N be a normal subgroup of group 'g' and the set

  $G/N = \{Na : a \in G\}$

  is collection of distinct right co-sets of N in G under multiplication the G/N is Quotient group or factor group.



Example : Let Z be an additive group of integers and let N be subgroup of Z defined by N = { nx | x ∈ Z}, where n is a fixed integer. Construct the quotient group Z/N. Also prepare a composition table for Z/N, when n=5.

Solution :-

An additive group Z of integer is abelian.

Then its subgroup N is a normal subgroup.

We have Z= {0, ±1, ±2,…}, the elements of a quotient group Z/N are the cosets which are as under.

N = {0, ±n, ±2n,…}

Now

N+0 = {0, 0±n, 0±2n,…}= N

N+1 = {1, 1±n, 1±2n,…}

N+2 = {2, 2±n, 2±2n,…}

….

….

N+(n-1) = {n-1, 2n-1,3n-1,-n-1,…}

similarly one can show that

$$N+(n+1) = N+1$$

$$N+(n+2) = N+2$$

i.e. $N+(n+i) = N+i \quad \forall\ i \in Z$

Hence $Z/N = \{ N, N+1, N+2, \ldots N+(n-1) \}$

For n=5: $N = \{0 \pm 5, \pm 10, \ldots\}$

The distinct cosets will be N,N+1,N+2,N+3,N+4. The composition table is

	N	N+1	N+2	N+3	N+4
N	N	N+1	N+2	N+3	N+4
N+1	N+1	N+2	N+3	N+4	N
N+2	N+2	N+3	N+4	N	N+1
N+3	N+3	N+4	N	N+1	N+2
N+4	N+4	N	N+1	N+2	N+3

Lagrange's Theorem

If G is a finite group and H is a subgroup of G, then o(H) is a divisor of o(G).

i.e. **o(G)/o(H)**

Example: If $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$ is a group of $H=\{a^3,$

$a^6 = e\}$ is its normal subgroup, then write G/H.

Solution : Here the distinct cosets of H in G are

$$eH = \{e.a^3, e.e\} = \{a^3, e\} = H$$
$$aH = \{a.a^3, a.e\} = \{a^4, a\}$$
$$a^2 H = \{a^2.a^3, a^2.e\} = \{a^5, a^2\}$$
$$a^3 H = \{a^3.a^3, a^3.e\} = \{a^6, a^3\} = \{e, a^3\} = H$$
$$a^4 H = \{a^4.a^3, a^4.e\} = \{a, a^4\} = aH$$
$$a^5 H = \{a^5.a^3, a^5.e\} = \{a^2, a^5\} = a^2H .$$

In this way we obtain only three distinct cosets H, aH, a^2H of H in G. Hence $G/N = \{H, aH, a^2H\}$.

Example : If G = < a >is a cyclic group of order 8, then the quotient group corresponding to the subgroup generated by a^2 and a^4 respectively.

Solution : Let, G= $\{a, a^2, a^3, a^4, a^5, a^6, a^7, a^8 = e\}$

and $H_1 = \{a^2, a^4, a^6, a^8 = e\}$

$H_2 = \{a^4, a^6, a^8 = e\}$

Since , G is abelian, therefore the subgroups H_1 and H_2 are normal in G.

$o(G/H_1) = 8/4 = 2,$

$o(G/H_2) = 8/2 = 4$

$G/H_1 = \{ H_1, H_1 a\}$, where $H_1 a = \{a^3, a^5, a^7, a\}$

$\{H_1 a^3 = H_1 a,\ \ H_1 a^2 = H_1 a^4 = H_1 a^6 = H_1 a^8 = H_1\}$ etc.

$G/H2 = \{H_2, H_2 a, H_2 a^2, H_2 a^3\}.$

THANK YOU